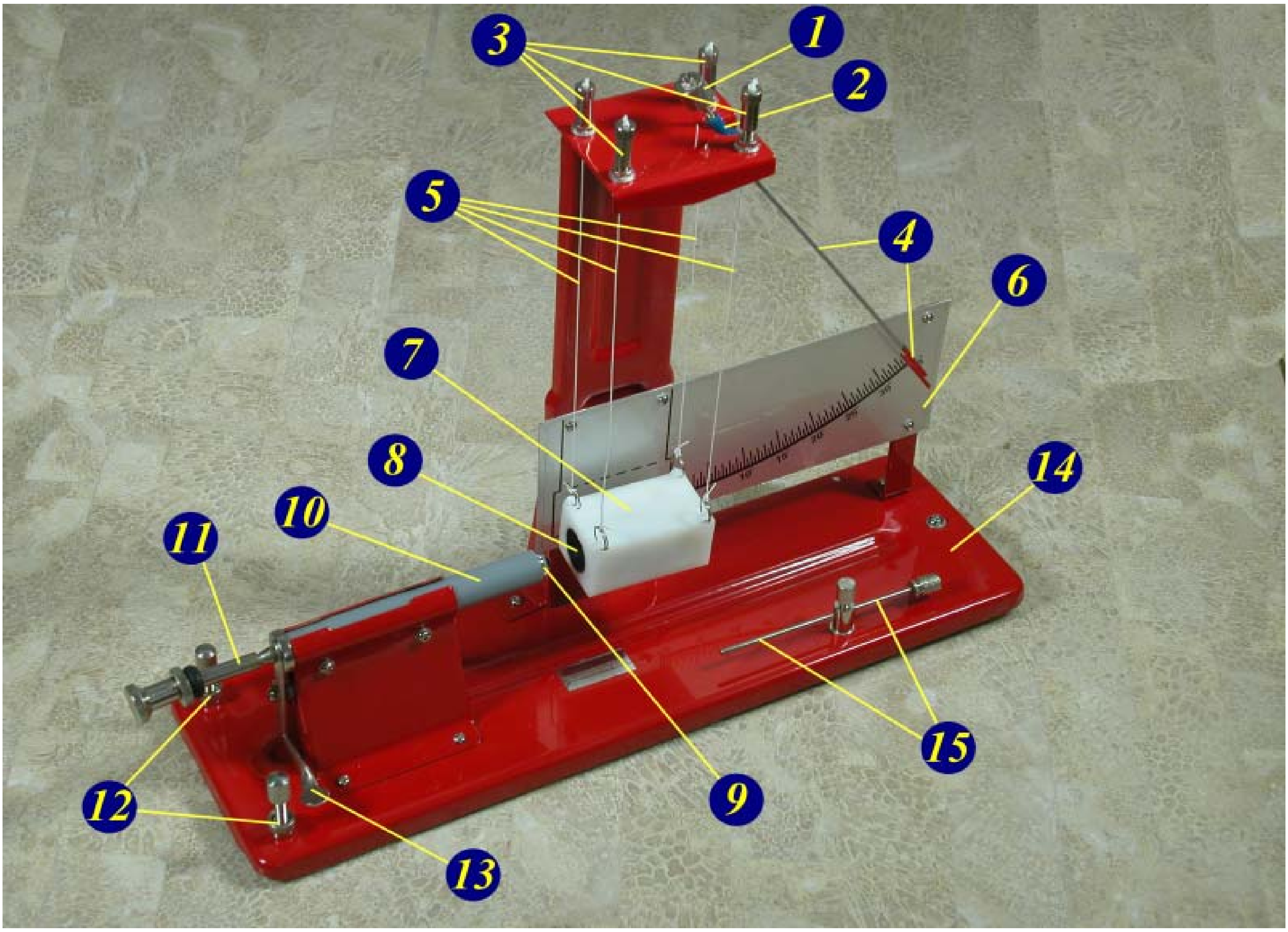

Introduction:

1. Balancer
2. Pressing Reed
3. Thread Regulator
4. Pointer
5. Pendulum Line
6. Scale Plate
7. Pendulum Block
8. Bullet Hole
9. Bullet
10. Gun Barrel
11. Breech Bolt
12. Levelling Screw
13. Trigger
14. Base Plate
15. Forking Rod

Operating Instructions for Ballistic Pendulum

1. Application:

This product is suitable for students to do the experiment of "measuring bullet speed" in the Physics Education in middle schools. In addition, it can be used for the experiments on "horizontal projectile motion".

2. Overview and Technical Data:

The unit consists of:

1). Spring gun:

It consists of a gun barrel, spring, breech bolt and a trigger, which are all fixed on the base plate. It can project "bullets" in three different speeds: V_1=5.4 ± 0.25 m/s, V_2=6.6 ± 0.25 m/s, V_3=7.7 ± 0.30 m/s.

2). Impact Pendulum:

Consisting of four pendulum lines (the length of four lines from hanging point to the upper surface of pendulum block is the same, i.e. 200 ± 2 mm), thread length regulators, frame and a pendulum block. The mass of the pendulum block is 80 ± 2 g. In the hole of the pendulum block damping materials were stuffed so that the impact of "bullet" against the pendulum block is non-elastic collision.

3). Indicator:

Consisting of a scale plate and pointer. The minimum angle of the scale is 0.5° and the maximum is 35° . The pointer is controlled by a balancing torque so that it can freely stay at any position.

4). "Bullet":

It is a steel ball with a mass of 7.5 ± 0.l g.

The percentage of hits for the ball to shoot into the Pendulum Block is not less than 95% if the unit has been correctly setup.
The total weight of the unit is about 1.65 kg.

3. Principle:

Because the declination θ of the pendulum block can be obtained from the experiment, we can calculate the bullet speed from the low of conservation of momentum and the principle of conservation of mechanical energy, i.e.

$$\frac{m_1 v_1^2}{2} = (m_1 + m_2)gh$$

The raised height of the pendulum block:

$$h = L(1 - \cos\alpha)$$

from which the bullet velocity may be obtained:

$$v_1 = \sqrt{\frac{2gL(m_1 + m_2)(1 - \cos\theta)}{m_1}}$$

in the formula, m_1 is the mass of bullet, m_2 is the mass of pendulum block, L is the length of pendulum and g is the gravity acceleration.

4. Procedures for Use:

A) To determine the velocity of the ball by use of the Conservation of Momentum and Energy

1. Level the base plate (use of the levelling adjustment screws).
2. Adjust the pendulum line screws and make the pendulum positioned just against the square block printed on the angle scale. The upper surface of the block should be parallel with the horizontal line on the angle scale. Move the pointer closely against the pendulum block so the pointer will show reading of 0° . This will ensure that the spring gun aims at the bullet hole in the pendulum block.
3. Pull out the breech bolt and position the trigger in one of the three existing grooves. Put the bullet into the gun's muzzle. Push down the trigger and the bullet will shoot into the pendulum block. This will produce a horizontal motion of the bullet and the block. This motion would push the pointer and it will deflect and consequently the bullet speed can be calculated and obtained from the deflection angle of the pointer. For reducing the errors the first reading from each different shooting position should be ignored. After the first shot, the pointer should be pulled back about 3°- 5° from its deflection position and then do the experiment again.

B) Use of the Pendulum as a projectile apparatus to determine the velocity of a ball when the ball had been set in horizontal projectile motion.

From the following formulas:

$$s = vt \qquad h = \frac{gt^2}{2}$$

We get::

$$t = \sqrt{\frac{2h}{g}} \quad \text{and} \quad v = \frac{s}{\sqrt{\frac{2h}{g}}}$$

Put the unit on a stable table. Remove the pendulum block from the unit and then turn the pointer clock wisely to the horizontal state. Spread sheets of paper on the ground and then trigger the “bullet”. Measure the distance "s" from gun's muzzle lo the point on the ground where bullet dropped onto and the height "h" of the muzzle. The initial bullet velocity can be calculated by use of the above formulas.

Compare the results obtained from type A or B experiments. The relative error designed for this unit is less than 12%.

Cautions:

1. The effective range of the spring gun is 3 metres.
2. If the turn of the pointer shows a too large or too small torque, the U-shaped pressing reed located at the pointer’s axle position can be adjusted (normally it had been factory adjusted).
3. The bullet should be oiled before storage if it will be out of use for a long time.

Advanced reading - Energy Definitions:

Nonrelativistically, the work done accelerating (or decelerating) a particle during the infinitesimal time interval *dt* is given by:

$$\mathbf{F}\cdot d\mathbf{x} = \mathbf{F}\cdot \mathbf{v}dt = \frac{d\mathbf{p}}{dt}\cdot \mathbf{v}dt = \mathbf{v}\cdot d\mathbf{p} = \mathbf{v}\cdot dm\mathbf{v} = \frac{m}{2}d(\mathbf{v}\cdot\mathbf{v}) = \frac{m}{2}dv^2 = d(\frac{mv^2}{2})$$

Since this is a total differential (that is, it only depends on the final state, not how the particle got there), we can integrate it and call the result Kinetic Energy:

$$E_k = \int \mathbf{F}\cdot d\mathbf{x} = \int \mathbf{v}\cdot d\mathbf{p} = \frac{mv^2}{2}$$

This equation states that the Kinetic Energy (E_k) is equal to the integral of the dot product of the velocity (**v**) of a body and the infinitesimal change of the body's momentum (**p**). It is assumed that the body starts with no Kinetic Energy when it is at rest (motionless).

Calculations:

There are several algorithms for calculating the Kinetic Energy of an object. The choice is determined by the velocity of the body or its size. Thus, if the object is moving at a velocity much smaller then the speed of light, the Newtonian (classical) mechanics is appropriate; but if the velocity is comparable to the speed of light, the rules of relativistic mechanics have to be followed. If the size of the object is sub-atomic, the quantum mechanical algorithm is most appropriate.

In Classical Mechanics, the Kinetic Energy of a "point object" (a body so small that its size can be ignored) is given by the equation $E_k = \frac{1}{2}mv^2$ where m is the mass and v is the speed of the body.

For example - one would calculate the Kinetic Energy of an 80 kg mass traveling at 18 meters per second (40 mph) as $\frac{1}{2}\cdot 80\cdot 18^2 = 12,960 \text{ joules}$.

Note that the Kinetic Energy increases with the square of the speed. This means, for example, that if you are travel twice as fast, you will have four times as much Kinetic Energy. As a result of this, a car traveling twice as fast requires four times as much distance to stop.

Thus, for non-relativistic mechanics, the Kinetic Energy can be calculated using the formula:

$$E_k = \frac{1}{2}mv^2$$

Calculation of Gravitational Potential Energy

Assuming that the opposing gravitational force is constant, the work done in raising an object is equal to the force applied multiplied by the distance through which the object is raised. The gravitational force that must be overcome is equal to the object's mass multiplied by the acceleration due to gravity, so the object's Gravitational Potential Energy, U_g, is given by

$$U_g = mgh$$

where

m is the mass of the object
g is the acceleration due to gravity (approximately 9.8 m/s^2 at the earth's surface)
h is the height to which the object is raised, relative to a given reference level (such as the earth's surface).

When applying this equation it is essential to use consistent units. Most scientific work is now done in SI units, in which case mass is measured in kilograms (kg), acceleration in meters per second squared (m/s^2), and distance (here height) in meters (m). The resulting energy is expressed in joules ($kg\ m^2/s^2$).

The equation shows that Gravitational Potential Energy is proportional to both mass and height. For example, raising two similar objects, or raising the same object twice as far, doubles the potential energy.

The "*mgh*" formula works well provided that the acceleration due to gravity, g, is very nearly constant over the distance h. On or close to the surface of the earth this assumption is reasonable, but over the much larger distances applying, for example, to spacecraft and astronomical bodies, it is not.

To calculate Gravitational Potential Energy with varying g it is necessary to sum all the individual increments of potential energy as the masses are separated, taking account of the varying value of g as we go. In the limit, as the increments become "infinitely small", the sum becomes an integral.

To simplify the evaluation of the integral we can make the assumption that the gravitational forces act as if the objects' masses were concentrated at their respective centers of mass. This assumption is mathematically exactly correct for a spherically symmetrical object (such as, to a reasonable approximation, a planet). It is *not* generally correct in other cases, though if the dimensions of an object are very small compared to the distance of separation then it is reasonable to consider it as a point mass and ignore the details of its shape.

With this simplifying assumption, integrating force over distance leads to the following general expression for the Gravitational Potential Energy, U_g, of a system of two masses:

$$U_g = \int_{h_1}^{h_2} \frac{Gm_1m_2}{r^2} dr$$
$$= Gm_1m_2 \left(\frac{1}{h_1} - \frac{1}{h_2} \right)$$

where

m_1 and m_2 are the masses of the two objects
G is the gravitational constant, $(6.6742 \pm 0.0010) \times 10^{-11}\ \mathrm{m}^3\ \mathrm{s}^{-2}\ \mathrm{kg}^{-1}$, (not to be confused with the g used earlier)
h_1 is the reference level (the separation at which potential energy is considered to be zero)
h_2 is the actual distance between the objects.

Subject to the caveats mentioned above, the distances h_1 and h_2 are measured between the objects' centres of mass.

For example, in the case of a small object above the surface of the earth, with reference level at the surface, m_1 and m_2 are respectively the masses of the earth and the object, h_1 is the distance from the earth's centre to the earth's surface, and h_2 is the distance from the earth's centre to the object.

If we try to calculate an "absolute" potential energy by setting the reference level at zero then the formula "blows up" with division by zero. In other words, we can only actually use this formula to measure the *difference* in potential energy between one non-zero separation and another.

In practice it is often convenient to take the reference level at infinite separation (i.e. $h_1 = \infty$), in which case the formula becomes:

$$U_g = \frac{-Gm_1m_2}{r}$$

where r is now the distance between the centres of mass of the two objects (again noting the earlier caveats). For a small object above the surface of the earth, r is the distance from the object to the earth's centre (and similarly for other spherical bodies).

Using this convention, potential energy is zero when r is infinitely large, and negative at any finite r. However, the *difference* in potential energy at different values of r – the quantity we are actually interested in – takes the expected sign.

U_g as calculated above measures the potential energy of the whole system. This can be visualised as if two bodies in space were released from rest and allowed to come together under the force of gravity. The sum of the Kinetic Energy gained by the two objects is exactly equal to the decrease in the potential energy of the system. The ratio of the objects' individual Kinetic Energy gains is equal to the reciprocal of the ratio of their masses. So, in the case of a relatively light object falling towards a very massive object (such as the earth), the contribution from the massive object is insignificant. In some sense, therefore, we can say that almost all the potential energy of the system is embodied in the light object, and almost none in the very massive object.