



Serrata Helmholtz Coils

The term **Helmholtz coils** refers to a device for producing a region of nearly uniform magnetic field. It is named in honour of the German physicist Hermann von Helmholtz.

Description

A Helmholtz pair consists of two identical circular magnetic coils that are placed symmetrically one on each side of the experimental area along a common axis, and separated by a distance h equal to the radius R of the coil. Each coil carries an equal electrical current flowing in the same direction.

Setting $h = R$, which is what defines a Helmholtz pair, minimizes the non uniformity of the field at the centre of the coils, in the sense of setting $d^2B / dx^2 = 0$, but leaves about 6% variation in field strength between the centre and the planes of the coils. A slightly larger value of h reduces the difference in field between the centre and the planes of the coils, at the expense of worsening the field's uniformity in the region near the centre, as measured by d^2B / dx^2 . [1]

Mathematics

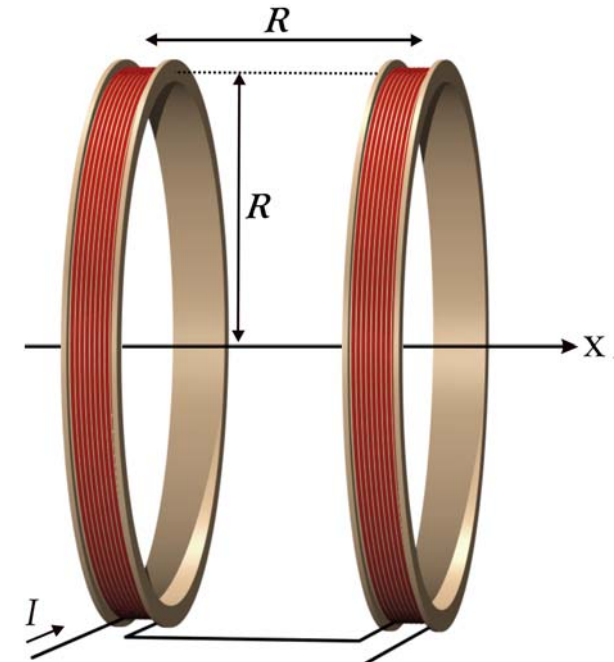
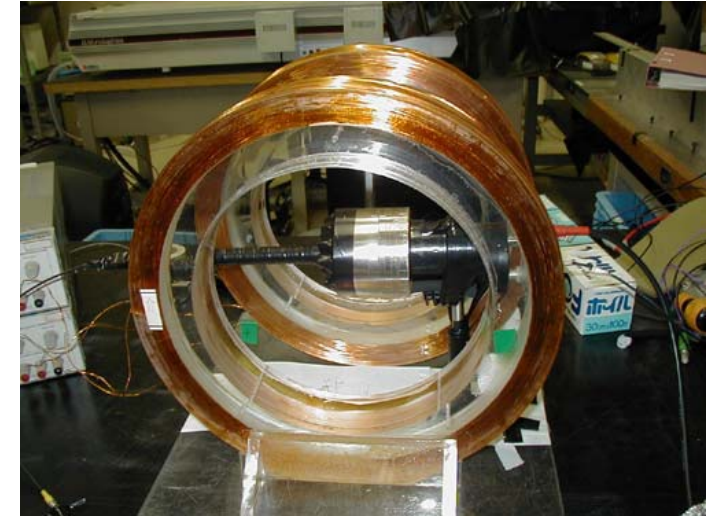
The calculation of the exact magnetic field at any point in space has mathematical complexities and involves the study of Bessel functions. Things are simpler along the axis of the coil-pair, and it is convenient to think about the Taylor series expansion of the field strength as a function of x , the distance from the central point of the coil-pair along the axis. By symmetry the odd order terms in the expansion are zero. By separating the coils so that $x = 0$ is an inflection point for each coil separately we can guarantee that the order x^2 term is also zero, and hence the leading non-uniform term is of order x^4 . One can easily show that the inflection point for a simple coil is $R / 2$ from the coil centre along the axis; hence the location of each coil at:

$$x = \pm R/2$$

A simple calculation gives the correct value of the field at the centre point. If the radius is R , the number of turns in each coil is n and the current flowing through the coils is I , then the magnetic flux density, B at the midpoint between the coils will be given by :

$$B = \left(\frac{4}{5} \right)^{3/2} \frac{\mu_0 n I}{R}$$

μ_0 is the permeability constant ($1.26 \times 10^{-6} \text{ T m/A}$) and R is in meters.



Helmholtz coil schematic drawing

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Derivation

Start with the formula for the on-axis field due to a single wire loop (which is itself derived from the Biot-Savart law):

$$B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

Where:

μ_0 = the permeability constant = $4\pi \times 10^{-7} \text{ T m/A} = 1.26 \times 10^{-6} \text{ T m/A}$

I = coil current, in amperes

R = coil radius, in meters

x = coil distance, on axis, to point, in meters

However the coil consists of a number of wire loops, the total current in the coil is given by

$$nI = \text{total current}$$

Where:

n = number of wire loops in one coil

Adding this to the formula:

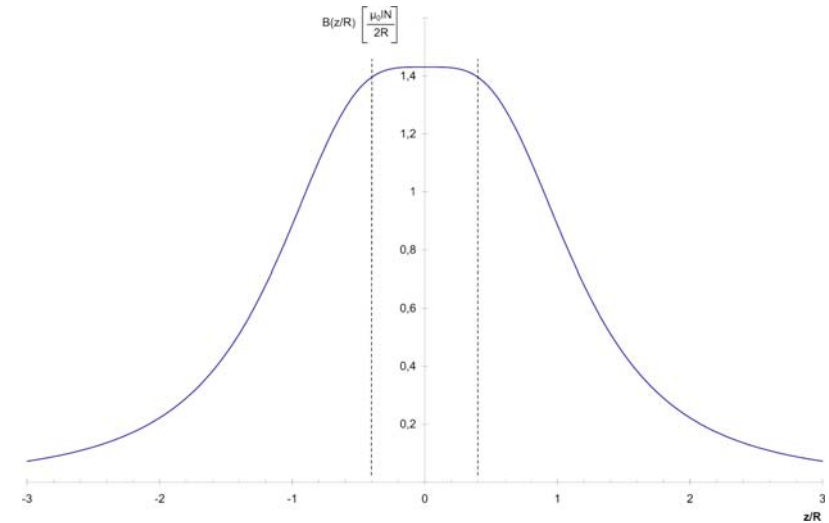
$$B = \frac{\mu_0 n I R^2}{2(R^2 + x^2)^{3/2}}$$

In a Helmholtz coil, a point halfway between the two loops has an x value equal to $R/2$, so let's perform that substitution:

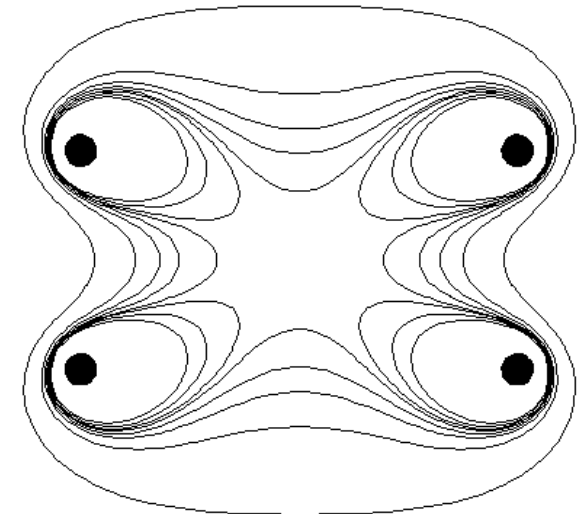
$$B = \frac{\mu_0 n I R^2}{2(R^2 + (R/2)^2)^{3/2}}$$

There are also two coils instead of one, so let's multiply the formula by 2, then simplify the formula:

$$B = \frac{2\mu_0 n I R^2}{2(R^2 + (R/2)^2)^{3/2}} \longrightarrow B = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 n I}{R}$$



Magnetic field induction along the axis crossing the centre of coils; $z=0$ is the point in the middle of distance between coils.



Contours showing the magnitude of the magnetic field near the coil pair. Inside the central 'octopus' the field is within 1% of its central value B_0 . The five contours are for field magnitudes of $.5B_0$, $.8B_0$, $.9B_0$, $0.95B_0$, and $.99B_0$.